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Inverse Design of Super-Elliptic Cooling Passages in Coated Turbine Blade Airfoils

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A highly accurate and reliable algorithm capable of performing automatic inverse design of coolant flow passage numbers, shapes, sizes, and locations inside coated solid objects has been developed. The user has the freedom to specify arbitrary temperatures and heat fluxes at the points on the outer surface of the object, and either temperatures or heat fluxes on the surfaces of the yet unknown coolant flow passages. The number of passages required could be guessed and the algorithm will automatically eliminate the unnecessary passages. The method allows even inexperienced designers to achieve an optimal configuration of coolant passages in a single computer run while satisfying user-specified manufacturing constraints that were incorporated via a barrier function method. The optimization algorithms used in this inverse design code were based on gradient search and on a modified Newton search. A simple method for escaping from local minima has been implemented that involves switching between two different formulations of the objective function. The optimal value of the gradient search parameter was found using a simple method of fitting a highly accurate spline through a set of points in the cost function/search parameter plane, and seeking out the value that will generate minimal error.

I. Introduction

THE design of internally cooled turbine blades is usually accomplished using approximate empirical methods, repetitive numerical analysis of intuitively modified coolant passage configurations, and expensive experimentation. In this approach, the proper number, dimensions, shapes, and locations of the coolant passages (holes) are usually determined according to the designer's experience. The development of high-speed computers and adequate numerical techniques has made it possible to approach the design problem differently and to solve it more efficiently and accurately.

The mathematical model for steady heat conduction in coated turbine blades is represented by the boundary value problem for Laplace's equation over a multiply-connected domain. For a given two-dimensional outer boundary shape, the temperature is specified on both the inner (hole) and outer (blade hot surface) boundary as is the initial guess for the number, locations, shapes, and sizes of coolant holes inside the blade. The goal is to determine the minimum necessary number and the proper locations, shapes, and sizes of the holes, such that the relative error between the specified and computed heat fluxes at the outer boundary is minimized. Specifying heat fluxes on the outer (hot) surface in addition to the already specified temperature distribution along the same surface creates an over-specified boundary value problem. Such an overspecified problem can be solved by an inverse (design) approach where the number, locations, shapes, and sizes of the holes are readjusted appropriately. These parameters can be determined using an optimization technique. The first application of such inverse methodology and optimization techniques in solving the design problem for turbine blade coolant flow passages was reported by Kennon and Dulikravich.[1–3] Their objective function was defined as a relative error between the specified heat fluxes on the outer (hot) boundary and the computed heat fluxes obtained by analyzing the temperature field for the current configuration of holes. Since an analytical solution to the boundary value problem for Laplace's equation governing a steady temperature field in an irregular multiply-connected domain does not exist, the boundary element method (BEM)[4] was used for its numerical integration. The BEM was chosen due to its simplicity, accuracy, and geometric flexibility and because it requires substantially less computing time than the finite element method or the finite difference method. Specifically, it does not require a repetitive costly two-dimensional computational grid generation for multiply-connected domains. The objective function was minimized using the Fletcher-Reeves gradient search method.[5]

Since then, this inverse design procedure for the optimization of coolant flow passages in turbine blades has been constantly improved. For example, inverse design of the passages in blades with ceramic coating[6] implicitly guarantees minimal temperature gradients throughout the structure. In addition to the specified heat flux on the outer surface, the turbine blade design was also made to satisfy two manufacturing constraints: 1) minimum allowable distance between neighboring holes as well as 2) minimum allowable distance between any hole and the outer boundary (or blade/coating interface if the blade is coated). A summary of methodologies for inverse (design) problems in steady and unsteady heat conduction was published by Dulikravich.[7]

More recently,[8] the inverse (design) problem of minimizing the number of necessary coolant flow passages having circular cross sections in addition to their proper dimensions (radii) and locations when temperature is specified on the outer (hot) surface and on the unknown, inner (cold) surface has been accomplished. Cross sections of the coolant flow passages were chosen to be circular since, from the manufacturing point of view, circular shapes are more desirable than arbitrary shapes.[1–3] This effort was then extended[9–11] to allow for coolant flow passages having cross sections that belong to a family of inclined Lame curves (superelliptic functions) of the general form:

$$\{[(x \cos \theta + y \sin \theta) - x_0]/a\}^n + \{[(y \cos \theta - x \sin \theta) - y_0]/b\}^n = 1 \quad (1)$$

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Here, global Cartesian coordinates x_0 and y_0 of the center of a superelliptic hole, hole semiaxis a and b, Lame curve exponent n, and the angle θ of inclination of the local inclined coordinate system x', y' with respect to the global coordinate system x, y, could be used as design variables.[9-11] This formulation provides more control over the shape of the hole since the exponent n allows for shapes varying smoothly from star-shaped ($n < 1$), to circular ($a = b$, $n = 2$) or elliptic ($a > b$, $n = 2$), to square or rectangular ($n \to \infty$). Thus, the shapes of the holes can vary significantly, while requiring only six parameters (x_0, y_0, a, b, n, θ) per hole to be optimized. It should be noted that the shapes of the holes should be defined not only from the manufacturing point of view, or by the requirements of heat conduction through the turbine blade, but also by the heat convection inside the coolant passage. This topic requires a fully three-dimensional extension of this design methodology and will not be addressed in the present work.

II. Optimization Technique

The complexity of the given inverse (design) problem requires the use of a relatively simple but robust and fast optimization technique for constrained nonlinear optimization. The Davidon-Fletcher-Powell (DFP)[5,12] algorithm combined with the barrier function method[12] was implemented[6-11] because it requires a relatively low number of objective function evaluations, which is the most expensive part of optimization in terms of computing time. The DFP method approximates the Hessian matrix so that second-order partial derivatives do not have to be computed directly. Using the barrier function method it is possible to include constraints in the objective function so that constraint functions and their gradients do not have to be computed. This approach, though, can cause convergence to become quite slow. A first-order numerical approximation (first-order accurate, one-sided finite differencing) was used to compute gradients of the objective function.

Since the number of holes is a design variable, the problem arises from the fact that this is the only discrete variable in the design vector. It is not a regular design variable because each of the initially guessed holes can be overspecified, and when computing the gradient of the objective function all the possible combinations should be taken into account. The most general algorithm should allow for increasing the number of holes as well. The question is where such new holes should be introduced and what should be their configurations. A simpler and more straightforward approach is to start optimizing with the highest possible number of holes (which is limited by the computer memory available), and then reduce the number of holes during the optimization procedure to a number which minimizes the objective function and satisfies all the constraints. The criterion for excluding a particular hole or holes from further optimization, and thus reducing the design vector by six design variables per each excluded hole, must be specified by the user. It can be defined as a percentage of the characteristic dimension of the problem. For example, it can be a small fraction of the chord length in the case of a turbine blade. Obviously, when the size of a hole becomes very small, its influence on the solution also becomes negligible. Instead of wasting computational time on recomputing the entire solution, we can eliminate any hole that is reduced beyond a prespecified size. Otherwise, the procedure is time consuming, it often terminates in a local minimum, and the obtained solution is not fully converged.[8]

A. Objective Function Switching

Optimization techniques require the highest possible accuracy of the analysis computer code.[6] Due to local minima that can occur in the objective function and due to truncation error, the code sometimes stalls before achieving an optimal solution. In order to overcome such a situation, a new technique has been devised.[8] In this approach, whenever the optimization stalls, the formulation of the objective function is automatically changed. The new objective function provides a departure from a local minimum and further convergence towards the global minimum.

Specifically, the objective to minimize the difference between the specified and the calculated heat flux at the outer boundary can be expressed in more than one way. In this work two different definitions of the objective function were used. The difference between the specified heat flux and the heat flux obtained from the current configuration is mathematically formulated as a L_2 norm. The objective function F can be computed as a global error (error in total heat flux through the blade outer boundary)

$$F_1(x) = \frac{\sum_{j=1}^{N_{\text{hot}}} (q_{\text{hot}}^{\text{spec}} - q_{\text{hot}}^{\text{calc}})_j^2}{\sum_{j=1}^{N_{\text{hot}}} (q_{\text{hot}}^{\text{spec}})_j^2 + \varepsilon} \times 100 \qquad (2)$$

or as a local normalized error in the heat flux at each panel j of the hot surface

$$F_2(x) = \sum_{j=1}^{N_{\text{hot}}} \left[\frac{(q_{\text{hot}}^{\text{spec}} - q_{\text{hot}}^{\text{calc}})^2}{(q_{\text{hot}}^{\text{spec}})^2 + \varepsilon} \right]_j \times 100 \qquad (3)$$

Here, N_{hot} is the total number of panels on the blade hot (outer) surface, $q_{\text{hot}}^{\text{spec}}$ is the specified (desired) heat flux, $q_{\text{hot}}^{\text{calc}}$ is the computed heat flux at the outer (hot) surface of the blade, and ε is a small user-specified parameter. For example, we used $\varepsilon = 1.0e - 6$.

B. Constraints and Barrier Function

The two manufacturing constraints were incorporated into the objective function using a barrier function of the following form:

$$B[g(x)] = CF_n(x) \sum_{m=1}^{M} \sum_{i=1}^{N_m} \left[\sum_{k=1}^{N_k} \frac{d^s}{(D_{ik}^s - d^s)} + \sum_{l=1}^{N_l} \frac{d^h}{(D_{il}^h - d^h)} \right] \qquad (4)$$

Here, M is the total number of holes, N_m is the number of panels on hole m, N_k is the number of panels on the interface between the blade and the coating, and N_l is the number of panels on the hole l. Here, D_{ik}^s is the current distance between the panel i on the hole m and the panel k on the interface between the core of the blade and the blade coating. Similarly, D_{il}^h is the current distance between the panel i on the hole m and the panel l on a different hole. The weighting function $CF_n(x)$ is initially large, and then gradually decreases so that with each optimization cycle the influence of the barrier function decreases. Here, the parameter $C < 1$ is a user specified value. The barrier function is established so that it has a singular value when the constraints are violated, and it is a rapidly growing function as long as the imposed constraints are nearly violated. It should be emphasized that every element in the sum given by the above formula [Eq. (4)] must be non-negative. If any element becomes negative, this means violation of the constraints. In that case the optimization process is stopped and computation of gradients is restarted using new spatial differences Δx and Δy. It was possible to use the barrier function in this problem since nonequality-type constraints are involved. The most important advantage of the barrier function is that it incorporates the constraints in the objective function. Thus, it always iterates through the feasible region and gives a feasible design. Finally, the composite objective function can have two forms

$$\tilde{F}_n[g(x)] = F_n(x) + B[g(x)] \qquad n = 1, 2 \qquad (5)$$

depending whether a global or a local objective function is used for its evaluation.

C. Optimum Line Search Parameter

It has been observed[7] that these types of optimization problems often have minimas of a "crevice" type, i.e., the global minima are hard to detect because they occur over a very narrow range of line search parameters α. Moreover, the optimal values of α often border on values of α that give a sharp rise in the composite error (objective function). Standard techniques (golden section, Fibonacci method, quadratic or cubic fitting, etc.) often do not work well or not at all[13] on these types of problems with narrow minimas. Consequently, we have used here a method described by Dulikravich[7] that is based on spline fitting and interpolation. In particular, composite objective functions for, e.g., six equally spaced values of α in the interval $0 < \alpha < 1$ are initially computed. A highly accurate exponential spline is then numerically fitted through these values of $\tilde{F}_n[g(x)]$ and interpolated at, e.g., 1000 equidistantly spaced values of α in the same interval $0 < \alpha < 1$. A simple search is then conducted to find α^*, i.e., an optimum value of α that will produce a minimum value of $\tilde{F}_n[g(x)]$ during the next optimization cycle. After α^* is actually implemented in the line search DFP algorithm, it will produce a certain value of the composite error, $\tilde{F}_n[g(x)]$. This value will be now added to the initial set of six points, therefore creating a more complete set consisting of seven $\tilde{F}_n[g(x)]$, α pairs. The exponential spline will now be fitted through the original six points plus this new point, and then interpolated at 1000 equidistantly spaced values of α in the interval $0 < \alpha < 1$. A new, improved value of α^* will be determined using a simple search and the entire procedure will be repeated. Typically, it suffices to use about 10 such cycles before the change in composite objective functions resulting from two consecutively evaluated α^* becomes negligible, indicating a truly optimal α^* for this optimization cycle.

In summary, the optimization procedure consists of the following steps:

1) Specify the shape of the outer (hot) surface and the coating of the turbine blade.

2) Specify the desired temperature distribution on the outer (hot) surface and the inner (hole) surfaces.

3) Specify the desired heat flux distribution on the outer (hot) surface. In practice, hot surface heat flux is obtained as a by-product of the hot flowfield computations around the blade when its hot surface temperature is specified.

4) Specify manufacturing constraints: a) minimum distance between holes and the metal/coating interface, and b) minimum distance between any two holes.

5) Specify an initial guess for the number of holes (M), their sizes (a_m, b_m), shapes (n_m), locations of their centers (x_{0m}, y_{0m}), and their inclinations (θ_m). Thus, if there are M holes, there will be $6 \times M$ design variables.

6) Using the BEM, the Dirichlet boundary value problem (specified temperatures on the blade outer surface and on the surfaces of the guessed holes) for Laplace's equation is solved. Heat fluxes through the outer boundary are computed and composite objective functions are formed. The gradient of the composite objective function is computed using finite differences. Laplace's equation is solved $6 \times M$ times, once for each design variable on each of the holes, in order to compute the gradient. To find a feasible design which minimizes the composite objective function in the direction of the gradient, different values of α, which multiplies the gradient of the composite objective function, have to be found. The value of α which gives the minimal value of $\tilde{F}_n$ is determined using repetitive exponential spline fitting and interpolation and then used to update the design variables.

7) Use the DFP technique to find the new values of design variables repeating the optimization procedure from step 6 until the variation of the corresponding composite objective function $\tilde{F}_n$ is below the value specified as the convergence criterion. If the dimension of one of the holes becomes less than the prespecified small value, the hole is eliminated from further optimization procedure. If the optimization procedure stalls in a local minimum, the objective function formulation is changed from Eq. (2) to Eq. (3) while continuing with optimization from step 6.

III. Boundary Element Formulation

It is assumed that the temperature field is already steady and that the solid material of the internally cooled configuration is thermally isotropic. Thermal expansion is neglected. Consequently, the governing energy equation is simply Laplace's equation

$$\nabla^2 T = 0 \qquad \text{in} \qquad \Omega \qquad (6)$$

In this work the BEM was applied. Instead of starting with the differential form of the boundary value problem for Laplace's equation and the boundary condition

$$\begin{aligned} T &= \bar{T} \qquad \text{on} \qquad \Gamma_u \\ q &= \bar{q} \qquad \text{on} \qquad \Gamma_q \end{aligned} \qquad (7)$$

where ∇^2 represents Laplace's operator, Ω denotes domain, Γ_q is the boundary on which heat flux $\bar{q}$ is specified, and Γ_u is the boundary on which temperature $\bar{T}$ is specified. The starting point is the integral form

$$\int_\Omega \nabla^2(T - \bar{T})\bar{w}\, d\Omega = 0 \qquad (8)$$

where Laplace's equation is multiplied by a function $\bar{w}$, continuous up to the second order. It should be noted that the boundary conditions are now incorporated in the equation itself. After integrating by parts twice and introducing the fundamental solution T^* instead of the function $\bar{w}$, the weighted residual statement for this problem can be written as

$$\int_\Omega (\nabla^2 T)T^*\, d\Omega - \int_{\Gamma_q} (q - \bar{q})T^*\, d\Gamma + \int_{\Gamma_u} (T - \bar{T})q^*\, d\Gamma = 0 \qquad (9)$$

The fundamental solution for using heat source surface singularities is

$$T^* = \left(\frac{1}{2\pi}\right) \ell n \left(\frac{1}{r}\right) \qquad (10)$$

where r is the distance from the point of observation to the surface panel point under consideration. The derivative of the fundamental solution normal to the boundary is

$$q^* = \frac{\partial T^*}{\partial n} \qquad (11)$$

where n is the direction normal to the boundary. The integral equation can be discretized by dividing the boundary Γ into a series of N linear elements. After discretization of the boundary, an additional integration by parts and substitution for q^*, the final form of the discretized integral equation is obtained as

$$c_i T_i + \sum_{j=1}^{N} T q^* \, d\Gamma_j = \sum_{j=1}^{N} T^* q \, d\Gamma_j \quad (12)$$

where T_i is the temperature of the ith point on the boundary and

$$c_i = \frac{\theta_i}{2\pi} \quad (13)$$

Here, θ_i is the internal angle of the corner between two neighboring surface panels expressed in radians.

The distribution of T and q along each surface element can be arbitrary. For the sake of simplicity and efficiency, the use of locally linear variations is known to give a sufficiently accurate solution. Then the discretized integral equation takes the following form:

$$c_i T_i \sum_{k=1}^{2N} G_{ik} q_k - \sum_{l=1}^{N} \hat{H}_{il} T_l \quad (14)$$

where N is the total number of boundary element nodes and i is the running index of the surface nodes. Here, diagonals of the matrix H are computed implicitly[4] rather than by calculating the interior angles at the corners between the neighboring panels. Introducing

$$H_{ii} = -\sum_{l=1}^{N} \hat{H}_{il} \quad \text{if} \quad l \neq i \quad (15)$$

Eq. (14) can be written as

$$\sum_{l=1}^{N} H_{il} T_l = \sum_{k=1}^{2N} G_{ik} q_k \quad (16)$$

Once all the unknowns are placed on the left side to form a vector X and matrices H and G are rearranged to form a new matrix, this new system of linear algebraic equations can be solved for vector X by Gauss elimination

$$AX = B \quad (17)$$

The problem of possible nonuniqueness of the solution to integral formulation Eq. (9), assuming a fundamental solution of the form given by Eq. (10), can be avoided by scaling the entire domain so that the maximal dimension is less than one.[6]

IV. Results

A. Coated Hollow Disk

This test case was used to evaluate the accuracy of the BEM code with sequentially varying boundary conditions and to demonstrate the hole elimination capability. The geometry consisted of a circular disk (radius 1.2 m) with a centrally located circular hole (radius 0.5 m). The disk was coated with the coating occupying the region 0.9 m $< r <$ 1.2 m. The ratio of thermal conductivities of the coating and the disk core region was 1:5. The outer surface of the disk and the interface between the disk core and the coating were discretized using 36 equal-length flat panels, respectively. The wall of the centrally located circular hole was discretized using 20 equal-length flat panels. The BEM code was first run with a constant temperature $T_{\rm hot}^{\rm spec}$ = 100 K specified on the outer surface (radius 1.2 m), and a constant temperature $T_{\rm cold}^{\rm spec}$ = 50 K on the inner surface (a circular hole of radius 0.5 m). The computed heat fluxes on the outer and the inner surface were then used as Neuman boundary conditions on the parts of the outer boundary and the circular hole. Specifically, the boundary condition specified on the upper half of the outer circular surface was the uniform computed radial temperature gradient ($q_{\rm hot}^{\rm spec}$ = 103 K/m), while on the lower half of the outer circular surface a uniform original temperature ($T_{\rm hot}^{\rm spec}$ = 100 K) was enforced. At the same time, the computed constant radial temperature gradient ($q_{\rm cold}^{\rm spec}$ = -250 K/m) was specified on the left wall ($3\pi/2 > \theta > \pi/2$) of the centrally located circular hole, while the original uniform temperature ($T_{\rm cold}^{\rm spec}$ = 50 K) was specified on the right wall ($\pi/2 > \theta > -\pi/2$) of the hole. When the numerical results obtained with the BEM code for this sequentially varying boundary conditions were compared with the locally analytical solutions for the heat flux and temperature, the BEM routine was found to be highly accurate, it had only 0.03% error vs the analytic solution.

The first test case involving optimization was then created by introducing three holes (Fig. 1) instead of a single centrally located circular hole. The three holes of various superelliptic shapes (an ellipse, a square, and a rounded rectangle) were used as an initial guess for the configuration which in reality should have only a single centrally located circular hole (Fig. 2). Values of the design variables for the initial three holes are summarized in Table 1 where the origin x_0, y_0 of the global Cartesian coordinate system x, y is at the center of the disk.

The boundary conditions on the surfaces of the three guessed holes were; a constant radial temperature gradient $q_{\rm cold}^{\rm spec}$ = -250 K/m was specified on the left wall ($3\pi/2 > \theta > \pi/2$) of each hole, while a constant temperature $T_{\rm cold}^{\rm spec}$ = 50 K was

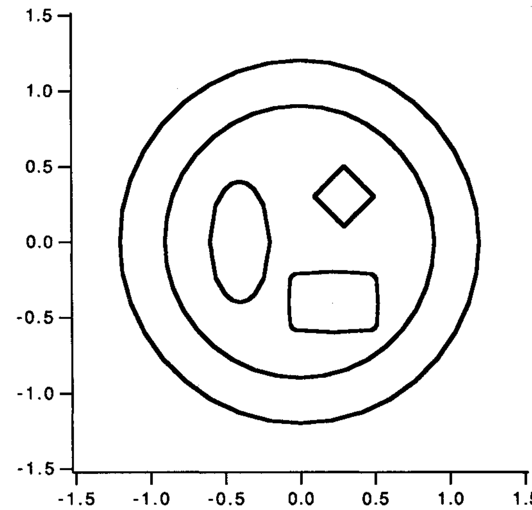

Fig. 1 Coated circular disk—initial configuration consisting of three superelliptic holes (a square, an ellipse, and a rounded rectangle).

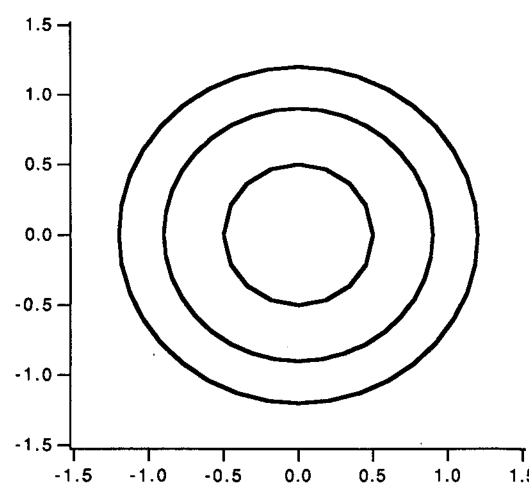

Fig. 2 Coated circular disk—correct target configuration.

Table 1 Case 1: Initial design variables

a	b	n	x_0	y_0	θ
0.4	0.2	2.0	−0.4	0.0	90
0.2	0.2	1.0	0.4	0.4	0
0.3	0.2	6.0	0.2	−0.4	0

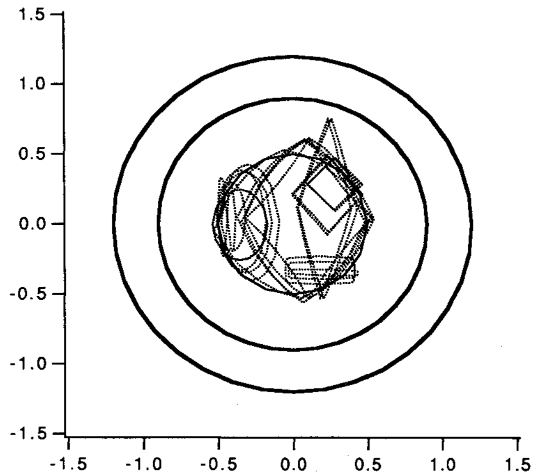

Fig. 3 Coated circular disk—final hole configuration (solid line) and their intermediate shapes (dotted lines). Two unnecessary holes (ellipse and a rectangle) were automatically reduced to zero.

specified on the right wall ($\pi/2 > \theta > -\pi/2$) of each hole. On the upper half of the outer disk surface a uniform temperature gradient ($q_{\rm hot}^{\rm spec}$ = 103 K/m) was specified, while enforcing the uniform temperature ($T_{\rm hot}^{\rm spec}$ = 100 K) on the lower half of the outer disk surface. Thus, the composite objective functions for the optimization procedure used in this test case were a combination of the composite objective functions based locally either on temperatures or on heat fluxes. The DFP optimization algorithm reached the fully converged solution (single, centrally located circular hole of 0.5-m radius) in 99 optimization cycles. It eliminated the third hole after 17 cycles and the first hole after 45 optimization cycles. Each hole was automatically eliminated when any of the two semimajor axis in the Lame representation of the hole geometry reduced below 1% of the disk radius. Similarly, the minimum allowable distance between any two holes or between any hole and the disk surface was specified as 1% of the disk radius. Figure 3 shows some of the intermediate hole configurations during the optimization process. The dotted lines indicate intermediate geometries, and the solid line is the final and fully converged geometry. Notice that the initially square-shaped hole dominated and eventually turned circular after the other two holes collapsed to needle-shaped objects. Figure 4 depicts the convergence history of the composite objective function where the spikes occur due to three reasons:

1) Automatic composite objective function switching from global to local [Eqs. (2) and (3)] and vice versa, since the local composite objective function is always larger than the global composite objective function.

2) Hole elimination causes a jump in the composite objective function. If the holes were allowed to shrink to an extremely small size, this jump would not occur. Nevertheless, the computer time for achieving this would be unjustifiably large.

3) The program is made to stop if the composite cost function switching is performed during two consecutive optimization cycles indicating the inability of the algorithm to escape the local minima. In this case, the design variables are user-perturbed and then the program resubmitted.

The entire optimization procedure required 2269 calls to the BEM analysis routine and consumed 2790 s of CPU time on an IBM 3090.

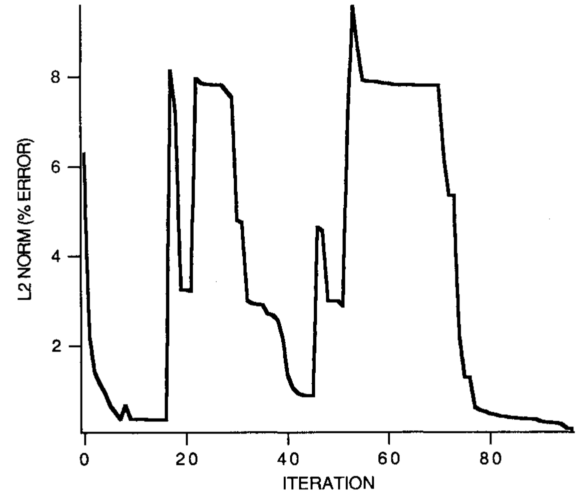

Fig. 4 Coated circular disk—convergence history (percentage of the integrated heat flux error on the outer boundary vs number of optimization cycles).

The apparently low error of the circumferentially integrated heat flux corresponding to the final converged solution (Fig. 4) was possible because the thermal conductivities of the metal disk and the ceramic coating were considerably different with the coating acting as a temperature gradient smoothing device.[8]

It should be pointed out that a similar test case consisting of a coated circular disk with an initial guess having 10 circular holes with constant temperatures specified also converged[8] to the correct solution represented by a single centrally located circular hole. This confirmed the insensitivity of the optimization algorithm to the number, shapes, sizes, and locations of the initially guessed holes and its reliability in detecting the global minimum.

B. Coated Turbine Blade Airfoil

We used a realistically shaped turbine blade airfoil having a chord length of 0.083 m and coating thickness 0.5% of the chord. The thermal conductivities were 1.0 W/m K for a typical ceramic coating material and 23.0 W/m K for the typical steel metal core material of the blade, thus making the ratio of thermal conductivities 1:23. It was assumed that the coated blade airfoil has three interior coolant flow passages and that their shapes, sizes, and locations are as depicted in Fig. 5, and in Table 2 where the values of a, b, x_0, and y_0 are given in meters. Here, the origin x_0, y_0 of the global Cartesian coordinate system x, y is at the geometric center of the blade airfoil.

All boundary conditions were of the Dirichlet type with a realistic variation of temperature specified on the outer (hot) surface (Fig. 5) and a constant temperature T = 500 K specified on the coolant flow passage walls. The fundamental solution for this composite domain problem was the same as given in Eq. (10). The outer (hot) surface of the blade airfoil was discretized with 50 flat panels. The metal/coating interface surface was also discretized with 50 flat panels. The surface of each of the three coolant flow passages (Fig. 5) was discretized with 20 flat panels. The panels were clustered everywhere with respect to the local surface curvature. In addition, a minimum distance of 0.0005 m between any of the holes and between any hole and the metal/coating interface were specified as the manufacturing constraints. The design variables for the fully converged geometry are listed in Table 3 and the target heat flux is shown in Fig. 6.

Figure 7 depicts the geometric evolution history at several stages during the optimization process. The optimization was completed when the normalized hot surface heat flux error (using the global cost function formulation and barrier function) reached 1.32%. The dotted shapes indicate the inter-

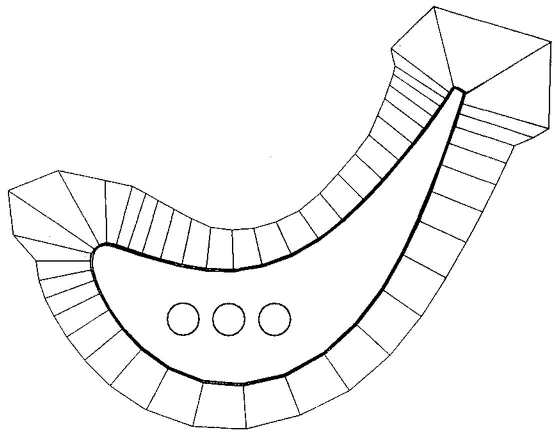

Fig. 5 Coated turbine blade with $T = 500$ K specified on each hole—specified temperature distribution on the blade outer (hot) surface and the initial hole configuration (three circles).

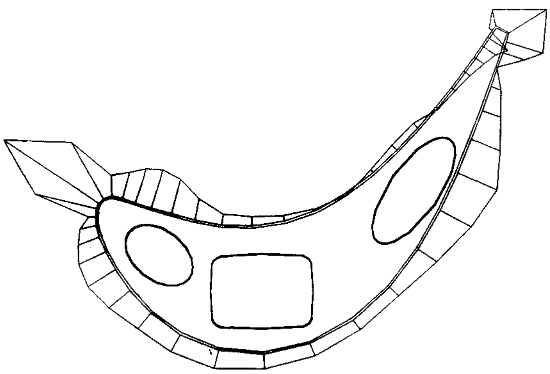

Fig. 6 Coated turbine blade airfoil with $T = 500$ K specified on each hole—specified heat flux distribution on the blade outer (hot) surface and target three-hole configuration. Outer surface temperature is specified as in Fig. 5.

Table 2 Case 2: Initial design variables

a	b	n	x_0	y_0	θ
0.005	0.005	2	−0.02	−0.0075	0
0.005	0.005	2	−0.01	−0.0075	0
0.005	0.005	2	0.00	−0.0075	0

Table 3 Case 2: Fully converged design variables

a	b	n	x_0	y_0	θ
0.0075	0.0050	2	−0.0275	−0.0025	−40
0.0100	0.0075	6	−0.0075	−0.0100	0
0.0125	0.0050	2	0.0275	0.0100	55

mediate geometries and the solid shapes indicate the final solution (not fully converged). Notice that each of the initially circular three holes transformed its shape appropriately and moved from their initial positions to the almost correct target configuration, which would have been eventually reached by continuing the optimization process and further reducing the composite cost function. The entire optimization in this test case consumed 103 optimization cycles, 2859 calls to the BEM analysis routine, and 12,028 s of CPU time on an IBM 3090 computer.

C. Increased Coolant Temperature

Assume now that a thermal systems designer wishes to use warmer high-pressure air from the last stages of the high-pressure compressor to cool the blades. This means an in-

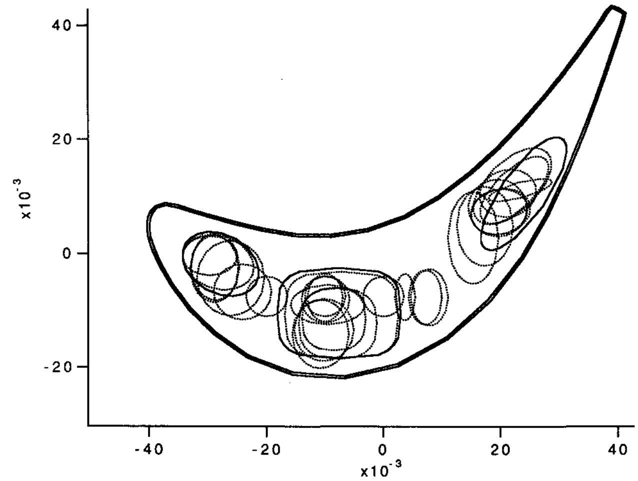

Fig. 7 Coated turbine blade airfoil with $T = 500$ K specified on each hole—geometric convergence history consisting of intermediate superelliptic hole shapes (dotted lines) and the converged configurations (solid lines). Outer surface thermal conditions as in Figs. 5 and 6.

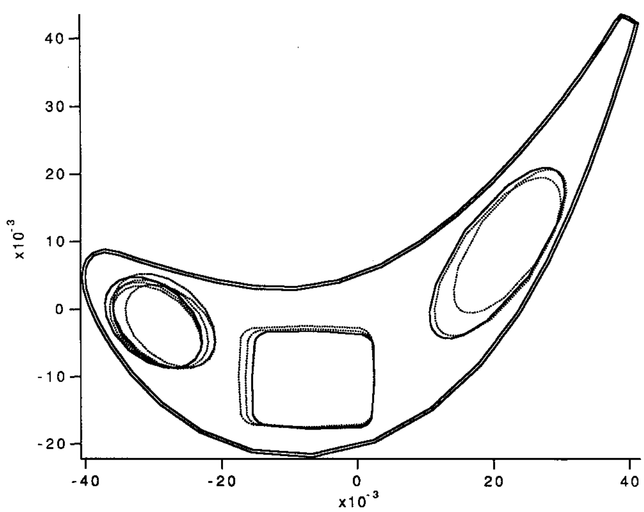

Fig. 8 Coated turbine blade airfoil with $T = 600$ K specified on each hole—intermediate shapes (dotted line) and the optimized configuration (full line) for the three holes. Outer surface thermal conditions as in Figs. 5 and 6.

crease in the temperature of the coolant passage walls from, e.g., 500 to 600 K. Also assume that the current values of the outer (hot) surface temperatures and heat fluxes are to be retained because of their direct influence on the aerodynamic performance of the blade. The objective is to examine if the previously optimized three-hole configuration (Fig. 6) with hole temperatures $T = 500$ K will be sufficient for the new, more extreme requirements of the holes with their surface temperatures $T = 600$ K. The initial guess (being the converged solution of the previous run with the hole temperatures $T = 500$ K) and the intermediate configurations of the holes are shown in Fig. 8 as dotted shapes, while the final shapes of the holes are depicted with solid lines. Figure 9 shows the convergence history of the optimization process, indicating that the process terminated at a local minima after the 12th optimization cycle. Composite objective function switching was performed automatically from global to local after the second cycle and vice versa after the eighth cycle. Notice that the local composite objective function was more effective than the global as the hot surface normalized heat flux error reduced only about 2% with the global formulation and 9% with the local formulation. Unfortunately, the global error in the hot surface heat flux only reduced from 32 to 30%, indicating that it is unlikely that a different constrained config-

Fig. 9 Coated turbine blade airfoil with $T = 600$ K specified on each hole—convergence history (percentage of the integrated heat flux error on the outer boundary vs number of optimization cycles). Outer surface thermal conditions as in Figs. 5 and 6.

uration with only three holes will allow for such a large increase in the temperature of the cooling fluid. In other words, the significantly warmer coolant in this instance requires a larger number of coolant flow passages in order to achieve the desired hot surface temperatures and heat fluxes.

V. Suggestions for Future Research

Future research in the field of inverse design of coolant flow passages in internally cooled configurations should be directed toward the development of multidisciplinary, complex design tools which will include fluid flow analysis involving thermal convection, radiation, and conduction, as well as thermal stress-deformation minimization while accounting for temperature-dependent thermal properties of the material.

The computer code based on the optimization algorithm presented here is not vectorized. Due to the nature of the problem and the BEM approach, which results in a large system of linear algebraic equations with fully populated matrices, it is not possible to fully vectorize the computational procedure in order to increase the computational speed. This problem can be solved by either employing different numerical methods to solve Laplace's equation or by implementing vectorization to the BEM. The latter goal can be achieved at the expense of a large amount of computer memory.

The same optimization procedure can be used for a variety[10] of practical problems governed by Laplace's partial differential equation. Faster and more reliable optimization packages can be readily substituted for the DFP routine. Specifically, we found that for the initial stages of the optimization process a quadratic programming algorithm of Pshenichny and Danilin[14] required considerably fewer iteration cycles and consumed only a third of the computer time[15] as compared to the DFP routine.

It is possible to approach an inverse design problem from a different point of view, besides the one presented in this work. A novel approach to the inverse design of coolant passages in turbine blades would be control theory approach. It has been shown recently that the control theory (adjoint operator)[16] approach to inverse design problems can efficiently lead to optimal solutions when optimizing a large number of variables. Finally, this method for inverse design is directly applicable and can be extended to fully three-dimensional[17] arbitrary configurations.

VI. Conclusions

An original technique for inverse design of two-dimensional coolant flow passages in arbitrarily-shaped coated objects using optimization has been extended and verified for the general class of superelliptic inclined cooling passage shapes. The optimization algorithm is reliable since it uses a novel approach to avoiding local minimas during the gradient search process. This shape design method is easy to use since it accepts practically arbitrary initial guesses for the number, locations, sizes, and shapes of the coolant flow holes. Unnecessary holes are automatically reduced to negligible size and eliminated.

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